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## *MATHEMATICS USED IN PHYSICS.*

*Einleitung in die Theoretische Physik.* Von Victor von Lange. Second Edition, Enlarged and Revised. (Braunschweig: Vieweg, 1891.)

THIS work is intended to give an account of the mathematical processes employed in physical investigations. It is divided into chapters dealing with the various branches of physics, mechanics, gravitation, magnetism, electricity, solids, fluids, gases, light, and heat. It is very difficult in such a book to decide how far to go in mathematical processes, and Herr von Lange has exercised his discretion wisely in this matter. At the other limit of how little to assume known he has certainly not erred in the direction of assuming too much, for he introduces proofs of simple differentiations and integrations when he requires them, which had much better be learnt continuously in an elementary treatise on the calculus. No English student would use a book of this advanced character without some preliminary mathematical training, and it is very doubtful whether anybody picking up the calculus in this haphazard fashion could ever use it in his own investigations; and if it is no use to him for this, would it not be a great saving of time and energy for him to depend on the investigations of others without going through all their work, just as an investigator of magnetic declination need hardly expect to have time to work through the lunar and planetary theories that help in the calculations of the *Nautical Almanac* he uses? A work of this kind is of great service as a concentrated store of information for those who want to study physics, and who have sufficient mathematical ability and training to be able to use the mathematical processes involved; but it cannot successfully compete with special treatises on the elements of solid geometry, differential calculus, &c., as a means of supplying the mathematical training required in order to use these processes.

Some readers may be disposed to doubt whether it is worth while introducing into a work of the scope of this book any elementary dynamics. The subject, however, wastes only a few pages, and it may very well be worth while introducing it in order to avoid references and explanations that might be quite as long. His discussion of the nature of mass is hardly satisfactory without a description of apparatus and methods of experimenting, but, so far as it goes, is fairly sound. He does not point out with sufficient clearness where definition ends and observation comes in. These are, however, really physical questions, with which a mathematical work might very well dispense. In discussing the rotation of a solid subject to forces, he bases his investigation on Airy's mathematical tracts, but he does not safeguard himself with all the provisos Airy so carefully introduces; and in consequence there are many pitfalls, carefully hidden. The method is based upon supposing the body given a series of blows, and appears on the face of it to be purely kinematical. It is on the other hand evident that, in general, dynamical questions, such as the centrifugal acceleration introduced when the axis of rotation is not a principal one, must come into consideration when discussing the forces that must be applied to a real body in order to make it move in a given way. A student of this investigation would be puzzled to understand how it happens that a solid sphere, when rotating round an axis and given a blow, begins to rotate round a new axis, new both inside the sphere, and in space, while a gyroscope takes up a wobble. It is possible by a series of blows given to a sphere to cause its axis of rotation to move round in space while preserving its position in the sphere, but a series of blows in general would not produce this result. The kinematic investigation of rotation of a solid round an axis accompanied by an angular acceleration round a rectangular axis is an interesting geometrical question, but must be carefully distinguished from the dynamical question of what forces must be applied to a real solid in order to produce this motion, and these two different questions not being sufficiently clearly distinguished make the investigation unsatisfactory. In connection with the motion of a solid, it is to be regretted that a short account of the theory of screws was not included.

Under gravitation at one place, there is a full account of free fall, pendulums, balances, bifilar suspensions, torsion balance, &c. Then he proceeds to questions depending on gravitation at different places, the figure of the earth, the constant of gravitation. Here he mentions Foucault's pendulum, and notices that the elementary investigation is insufficient, without, however, giving more than the result of the complete investigation, not even explaining why the elementary investigation fails, owing to the precessional motion of the axes of the ellipse in which the bob of the pendulum necessarily moves, and which becomes comparable with the motion looked for, unless the amplitude be very small and the suspending thread very long. This chapter concludes with an account of the theorems connected with forces varying inversely as the square of the distance. It is doubtful whether it would not have been better to deal with this subject in the first place from the hydrodynamical point of view. Such theorems as that the flow is equal across every section of a tube of flow, and its numerous consequences, such as that equal quantities of electricity exist at the ends of a tube of force, that the total normal force over any surface is equal to $4\pi$ times the quantity of electricity within, &c., are all intuitively evident in hydrodynamics, and it is well to call a student's attention to the way in which he can safely argue from the familiar to the unfamiliar.

The chapter on magnetism is very complete, though the action of two magnets on one another is done in a fearfully long-winded way; and in the account of the determination of magnetic declination the spherical trigonometry required in order to calculate the azimuth of the terrestrial meridian from the astronomical observations is not given. It would also appear as if the determination of variations of dip by means of an induction vertical force magnetometer were quite a different thing from determinations of the variation of vertical force by means of a balance magnetometer. Magnetic induction is the usual mathematical investigation of simple cases where the permeability is assumed constant. An

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edition dated 1891 might have included some of the mathematics of hysteresis.

Electrostatics is treated as fully as it should be, though perhaps a single chapter on the law of the inverse square, containing most of the theorems required in magnetism and electricity, would have given a sounder view of the mathematics involved. It was hardly to be expected that a mathematician should avoid the temptation of describing Mossotti's theory of dielectrics without a warning that it can hardly be complete, and in consequence gives the electrical displacement as

$$\frac{K-1}{4\pi}$$

instead of $\frac{K}{4\pi}$ times the electrical force, thus making the displacement zero in a vacuum, and justifying this by saying that the results differ very little, while it would really overturn the whole electro-magnetic theory of light. This same overturning is calmly got over when the electro-magnetic theory of light is considered further on by a reference to this place, and this very remarkable statement that $K-1$ differs but little from K. There seems to be some confusion, arising from the fact that in electro-magnetic measure K is nearly $10^{21}$, but such a muddle is inexcusable. He further on gives the theory of penetration of electric force into conductors, without referring back to an investigation he has previously given of the concentration of alternating currents on the surface of a wire, not appearing to appreciate that they are the same. He also actually explains wave propagation in dielectrics by induction from layer to layer because the inducing force is *very small* initially at a distance. He has not learnt the A B C of action by means of a medium, but is still hampered by the dry bones of theories of action at a distance. In consequence of this, his investigation of the magnetic action of electric currents is all bristling with the action of elements upon one another, and little or no attention given to the energy stored in the medium, or how it goes from place to place.

The chapter on solids begins with some rather doubtful physical paragraphs that are out of place in a mathematical work. Is it sound to call heat a force (*Kraft*) that holds the particles of bodies asunder? Is it sound to say that the difference between solids and liquids is the difficulty of separating the parts of the former, when it is known that it often takes hundreds of pounds per square inch to separate the parts of a liquid from one another, and when it is the resistance of the material to shear that he really uses as the characteristic of solids? That mistake of making the difficulty of separation and not the difficulty of shearing the characteristic of solids seems quite common: it occurs in many books. The mathematical theory of elasticity is given in the usual analytical way, and applied to some of the simpler cases of bending, &c. Periodic motion is then introduced, and the more important cases of wave motion and vibrations of solids considered. In the consideration of torsion he omits to give any warning as to difficulties arising in the case of non-cylindrical prisms. The chapter concludes with an investigation of the impacts of solid spheres in a manner that brings it into connection with the kinetic theory of gases.

The chapters on liquids and gases are fairly complete. There is an interesting numerical calculation of the height of a statical tide: this is an example of how complete and varied are the physical questions of which the mathematics is given by Herr von Lange. The elementary kinetic theory of gases is given, but without any discussion of the distribution of velocities amongst the molecules. Van der Waals's modification of the simple gaseous laws is discussed, and along with it the theory of cubic equations is given in a rather skimpy form—an example of how difficult it is to teach the higher physical mathematics in a way that applies to the particular case in hand, except by teaching the part of the higher mathematics involved from a wider point of view than the particular solution requires.

The chapter on light is hardly so full as such an important subject demands. Diffraction is run through, but the absence of bands inside a shadow is not discussed, and the theory of definition in telescopes is separated from the same question in microscopes in a very unscientific way. There is a lot of reflection theory, and a paragraph on the direction of the vibration relative to the plane of polarization; but no notice is taken of the theory of the blue sky, nor of the electro-magnetic method of determination, nor of Wiener's proof that it is the electric force which acts on silver salts, and is consequently the one probably effective in most chemical actions, and therefore in irritating the retina. It is possible, however that iron salts may be acted on by the magnetic force.

The last chapter is on conduction of heat and on the mechanical theory of heat. The first part is an account of the simpler parts of Fourier, as any book on conduction of heat must be, and the latter is a good account of thermodynamics. It is to be regretted that he does not give some mechanical illustrations of temperature, though a discussion of the nature of temperature would have been out of place. The chapter concludes with a variety of applications of thermodynamics to such questions as the relations of electromotive force, compressibility, and surface tension, to temperature, as well as the usual one, vapour pressure. It is much easier to point out defects than adequately to describe excellences. It must not therefore be concluded from the fact that much of this review is concerned with the former that the defects preponderate over the excellences of Herr von Lange's work. On the contrary, the work is full of excellences. The way in which physics and mathematics are tending to grow each purer—one in the direction of mathematical abstractions, complexes, matrices, and such like; the other in the direction of experimental methods, accuracy, phenomena, and such like—makes it daily more important for physical investigators especially to have by them a convenient *résumé* of those parts of mathematics that are most often useful to them in their investigations, and this has been ably supplied by Herr von Lange.

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## *PHASES OF ANIMAL LIFE.*

*Phases of Animal Life, Past and Present.* By R. Lydekker, B.A. (Cantab.). (London: Longmans and Co., 1892.)

THE sixteen essays which make up the volume are reprints, with a few alterations, of articles originally published in *Knowledge.* "They are intended," the

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